

## Grade 8 Fraction Multiplication

8.N.6	
<p>Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically.</p>	<ol style="list-style-type: none"> <li>1. Identify the operation required to solve a problem involving positive fractions. [DEVELOPED]</li> <li>2. Provide a context that requires the multiplying of two positive fractions. [DEVELOPED]</li> <li>3. Provide a context that requires the dividing of two positive fractions. [NOT DEVELOPED]</li> <li>4. Express a positive mixed number as an improper fraction and a positive fraction as a mixed number. [NOTE]</li> <li>5. Model multiplication of a positive fraction by a whole number, concretely or pictorially, and record the process. [DEVELOPED]</li> <li>6. Model multiplication of a positive fraction by a positive fraction, concretely or pictorially using an area model, and record the process. [DEVELOPED]</li> <li>7. Model division of a positive proper fraction by a whole number, concretely or pictorially, and record the process. [NOT DEVELOPED]</li> <li>8. Model pictorially division of a positive proper fraction by a positive proper fraction, and record the process. [NOT DEVELOPED]</li> <li>9. Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers. [DEVELOPED only for MULTIPLICATION]</li> <li>10. Solve a problem involving positive fractions taking into consideration order of operations (limited to problems with positive solutions). [DEVELOPED]</li> </ol>

### Clarification of the outcome:

- ◆ This outcome concerns understanding and being able to use efficient fraction multiplication algorithms.

### Required close-to-at-hand prior knowledge:

- ♣ Understands the part of a whole, part of a set/group, and measure meanings of fraction.
- ♣ Understands mixed numbers and fraction equivalency.

### NOTE: (Achievement indicator #4)

In grade 7, students are expected to add/subtract mixed numbers. That involves converting between mixed numbers and improper fractions. The assumption here is that students can convert. Therefore indicator #4 is not addressed.

## SET SCENE stage

### The problem task to present to students:

- *There is  $\frac{3}{4}$  of a pie left to eat. You have  $\frac{1}{2}$  of it for supper. What fraction of a whole pie did you eat for supper?*
- *I have ten booklets that are each  $\frac{1}{2}$  cm thick. I pile the booklets on top of each other. How thick is the pile?*

Organize students into groups. Ask each group to solve the problems in whatever way makes sense to them. Allow them freedom to explore solution paths.

### Comments:

The purpose of the task is present a situation that begins students' journey into fraction multiplication.

### Note:

There is an important matter related to teaching fraction multiplication. It concerns the nature of a group. For example, you can have one group of people, two groups of apples, and so on. You cannot have a fractional group count. For example, you cannot have a  $\frac{1}{2}$  group of people. The people in that so-called half group are one group of people, not a  $\frac{1}{2}$  group of people. However, you can figure out how many people in half of one group of 12 people. You can solve this problem by multiplying ( $\frac{1}{2} \times 12$ ) but the ' $\frac{1}{2}$ ' is not a group indicator. It is a comparison number that is similar to a scale factor. In other words, it is inappropriate to talk about the ' $\frac{1}{2}$ ' as a half group because a group must be a whole number (0 groups of 12, 1 group of 12, 2 groups of 12, etc.). This reality has significance for the kinds of activities that can be used to help students understand fraction multiplication. Activities that treat (for example)  $\frac{1}{2} \times 4$  as meaning a half a group of 4 are wandering into mathematical and language untruths.

## DEVELOP stage

### Activity 1: Revisits SET SCENE

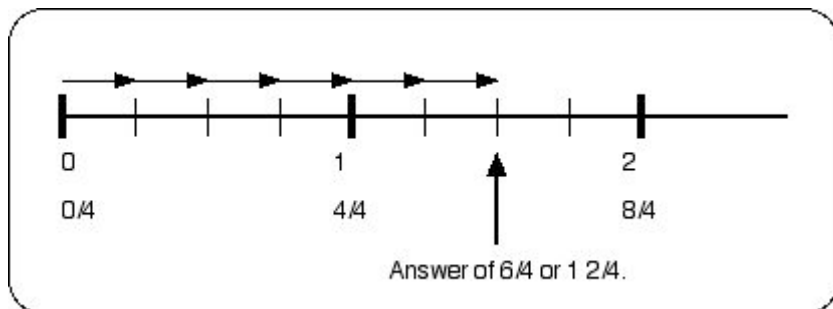
Ask selected groups to present their methods of solving and their solutions for the SET SCENE problems. Do not indicate correct/not correct. Just acknowledge the approaches.

Guide them to interpret both problems in a multiplication way ( $1/2 \times 3/4 = ?$  and  $10 \times 1/2 = ?$ ). This may be more difficult than you might imagine. It is likely students thought of the problems in a division way (e.g. split  $3/4$  into two equal parts; split 10 into 2 equal parts). Suggest that it is wiser to interpret the problems in a multiplication way because that makes it easier to solve similar problems. For example, you could discuss how division thinking might not be useful for solving the pie problem if you ate  $3/5$  of a  $3/4$  pie for supper.

Mention that students will first consider a whole number  $\times$  a fraction and a fraction  $\times$  whole number before considering a fraction  $\times$  a fraction.

### Activity 2: Addresses achievement indicators 1, 2 and 5.

- ◆ Pose a problem that involves a whole number  $\times$  a fraction and measurement (e. g. *You are stacking 6 boards on top of each other. Each board is  $1/4$  inches thick. How high will the stack be?*). Ask students to provide a multiplication number sentence that represents the situation (in this case,  $6 \times 1/4 = ?$ : 6 groups of  $1/4$ ). Ask students to use a number line to solve the problem. Expect them to take 6 steps of  $1/4$ , stopping at  $6/4$  or  $1 \frac{2}{4}$  (see diagram).



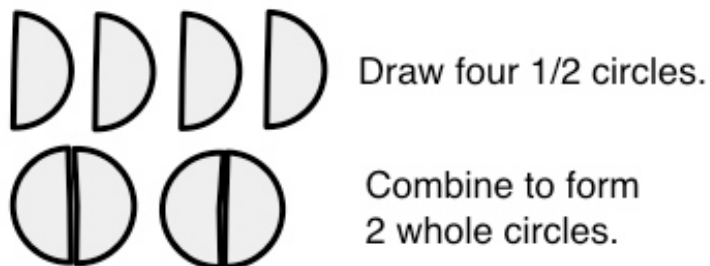
- ◆ Pose two other 'whole number  $\times$  proper fraction' measurement problems. Ask students to provide a multiplication number sentence that represents the situation. Ask them to solve the problems using a number line. Do not discuss a symbolic method for multiplying fractions yet.

#### Note:

If an answer is 1 or larger, always emphasize the fraction form (e.g.  $5/3$ ). Only acknowledge the mixed number form (e.g.  $1 \frac{2}{3}$ ). Do not reduce answers (e.g. If the answer is  $2/4$ , do not write  $1/2$ ; leave it as  $2/4$ ). Why? Unreduced fraction forms (e.g.  $8/4$ ) are the most useful for students discovering efficient fraction algorithms.

**Activity 3: Addresses achievement indicators 1, 2 and 5.**

- ◆ Pose a problem that involves a whole number  $\times$  a proper fraction involving the part of a whole meaning of fraction (e. g. *You eat 4 half pieces of pie. How many whole pies did you eat?*). Ask students to provide a multiplication number sentence that represents the situation (in this case,  $4 \times \frac{1}{2} = ?$ : 4 groups of  $\frac{1}{2}$ ). Ask students to use fraction circles/ fraction bars to do the arithmetic. In the case of fraction circles, expect them to make 4 groups of  $\frac{1}{2}$  circles and combine them to obtain 2 whole pies (see diagram).



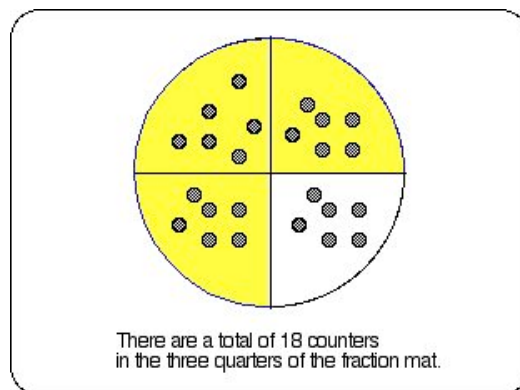
- ◆ Pose two other ‘whole number  $\times$  proper fraction’ problems involving the part of a whole meaning of fraction. Ask students to provide a multiplication number sentence that represents the situation. Ask them to solve the problems using fraction circles or fraction bars. Do not discuss an efficient method for multiplying fractions yet.

**Note:**

If an answer is 1 or larger, always emphasize the fraction form (e.g.  $\frac{5}{3}$ ). Only acknowledge the mixed number form (e.g.  $1 \frac{2}{3}$ ). Do not reduce answers (e.g. If the answer is  $\frac{2}{4}$ , do not write  $\frac{1}{2}$ ; leave it as  $\frac{2}{4}$ ).

**Activity 4: Addresses achievement indicators 1, 2 and 5.**

- ◆ Pose a problem that involves a proper fraction  $\times$  a whole number and the part of a group meaning of fraction (e.g. *Three quarters of the 24 students in the class like pizza. How many students like pizza?*). Ask students to provide a multiplication number sentence that represents the situation ( $\frac{3}{4} \times 24 = ?$ ). Ask students to draw a circle and use counters to obtain an answer. Expect them to use a circle divided into quarters, placing 6 objects in each quarter and then using three of them to determine the answer of 18 (see diagram).



**Note:**

The issue that, in  $\frac{3}{4} \times 24$ ,  $\frac{3}{4}$  cannot be a group count arises. It will need discussion.

**Note:**

By this point you should have enough data (fraction multiplication questions and answers) for students to use as a reference for determining an efficient method for multiplying a whole number  $\times$  a proper fraction. Activity #5 concerns that.

**Activity 5: Addresses achievement indicator 9 and practice.**

- ◆ Write on the board the list of fraction multiplication questions and answers that were involved in activities 2 through 4. Ask students to examine the list of fraction number sentences and then to determine a method for multiplying a whole number times a fraction or a fraction times a whole number. Assist as needed. Ensure that students realize that a method involves multiplying the whole number times the numerator (top) of the fraction and keeping its denominator (the bottom). [**NOTE:** Reduce answers as desired.]
- ◆ Provide six fraction multiplication questions (whole number  $\times$  proper fraction OR proper fraction  $\times$  whole number). Ask students to obtain answers, using the fraction multiplication method discussed.

**Activity 6: Addresses achievement indicator 9 and practice.**

- ◆ Discuss fraction names for whole numbers. For example, another name for 3 is  $\frac{6}{2}$ . Then focus on another name for a whole number where it is represented by a fraction having a denominator of 1 (e.g. 3 is  $\frac{3}{1}$ ).
- ◆ Revisit some of the fraction multiplication questions from previous activities and ask students to write the whole number involved as a fraction (e. g.  $4 \times \frac{1}{2}$  becomes  $\frac{4}{1} \times \frac{1}{2}$ ). Discuss whether the question has changed, even though it looks different. Discuss whether the answer should still be the same. Ask students to determine a method for multiplying  $\frac{4}{1} \times \frac{1}{2}$ . Assist as needed. Ensure they realize that the method involves numerators  $\times$  numerators and denominators  $\times$  denominators. [**Reducing is appropriate.**] Repeat this with several other previously completed fraction multiplication questions.
- ◆ Present the multiplication question  $\frac{1}{2} \times \frac{4}{5}$ . Ask students whether the answer should be smaller than  $\frac{4}{5}$ . Expect 'yes'. Ask students to use the method developed in activity #6 to obtain an answer to  $\frac{1}{2} \times \frac{4}{5}$ . Expect  $\frac{2}{5}$ . Discuss that the efficient method 'numerators  $\times$  numerators and denominators  $\times$  denominators' seems to work.
- ◆ Provide students with six multiplication questions that concern a proper fraction  $\times$  a proper fraction. Ask students to use the efficient method to obtain answers. Assist and discuss as needed.

**Activity 7: Addresses achievement indicators 1, 2, 5, and 9, and revisits SET SCENE.**

- ◆ Revisit the two problems from SET SCENE. Ask students to redo them using what they have learned to this point. Discuss their work.

**Activity 8: Addresses achievement indicators 1, 4, and 9.**

- ◆ When you are confident that students can multiply a proper fraction  $\times$  a proper fraction, using the efficient method, pose a problem involving a proper fraction  $\times$  a mixed number. (e.g. *You buy  $1\frac{3}{4}$  kg of butter. You use  $\frac{1}{2}$  the butter for baking. How much butter was used for baking?*) Ask students to represent the problem using a multiplication number sentence. Expect  $\frac{1}{2} \times 1\frac{3}{4} = ?$  or  $1\frac{3}{4} \times \frac{1}{2} = ?$ . Remind students about mixed numbers and fractions greater than 1 (improper fractions). Ask them to change the mixed number to a fraction greater than 1 and to obtain an answer using the efficient multiplication method.
- ◆ Provide five other questions involving fractions  $\times$  mixed numbers. Ask students to obtain the answers. Discuss their work.

**Activity 9: Addresses achievement indicator 10.**

- ◆ Present students with arithmetic tasks that involve addition/subtraction and multiplication of fractions (e.g.  $3\frac{1}{2} + \frac{3}{4} \times \frac{1}{2} - 1\frac{2}{3}$ ). Ask students to obtain answers, paying attention to the order of operations. Discuss their work.

**Activity 10: Assessment of teaching.**

- 🌐 Provide students with two simple problems (problems, not arithmetic questions), involving a proper fraction  $\times$  a proper fraction and one problem involving a proper fraction  $\times$  a mixed number. Ask students to solve the problems. Ask students to select one of the problems and obtain the answer by using a model (number line, fraction bars, etc.).

*If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).*

*An example of a partial well-designed worksheet follows.*

*The worksheet contains a sampling of question types. More questions of each type are needed.*

The MAINTAIN stage follows the sample worksheet.

### Question 1.

- a) Do the fraction multiplication question,  $4 \times \frac{2}{3}$ , by using repeated addition.
- b) Redo the question,  $4 \times \frac{2}{3}$ , by using a number line.
- c) Should the answers for part a) and part b) be the same?

### Question 2.

- a) Do the fraction multiplication question,  $2 \times \frac{3}{4}$ , by using repeated addition.
- b) Redo the question,  $2 \times \frac{3}{4}$ , by using fraction circles (or bars).
- c) Should the answers for part a) and part b) be the same?

### Question 3.

Obtain an answer to each fraction multiplication using an efficient method. DO NOT reduce.

- a)  $4 \times \frac{3}{4}$
- b)  $10 \times \frac{2}{5}$
- c)  $\frac{2}{3} \times 18$

### Question 4.

Obtain an answer to each fraction multiplication using an efficient method. DO NOT reduce.

- a)  $6 \times \frac{3}{7}$
- b)  $12 \times \frac{3}{8}$
- c)  $\frac{4}{5} \times 9$

### Question 5.

Obtain an answer to each fraction multiplication using an efficient method. Reduce if possible.

- a)  $\frac{1}{2} \times \frac{3}{4}$
- b)  $\frac{2}{3} \times \frac{4}{5}$
- c)  $\frac{3}{7} \times \frac{3}{10} \times \frac{1}{2}$

### Question 6.

Obtain an answer to each fraction multiplication using an efficient method. Reduce if possible.

- a)  $1 \frac{2}{3} \times \frac{3}{8}$
- b)  $\frac{4}{5} \times 2 \frac{1}{2}$
- c)  $2 \frac{1}{3} \times 3 \frac{2}{5}$

### Question 7.

Solve each of the following problems. Show your work.

- a) Mark had 4 lengths of string. Each string was  $\frac{2}{3}$  metres long. Mark combined the four lengths into a long string. How long would it be?
- b) Janice walks in order to keep fit. She walks  $\frac{9}{10}$  km each day. If she walked 12 days in a row, what would be the total distance she walked?
- c) Barb had  $\frac{3}{8}$  of a litre of orange juice in the fridge. She poured  $\frac{2}{3}$  of it into a glass to drink. How much orange juice did she pour into the glass?
- d) Jason bought  $2\frac{3}{4}$  kg of candies. He ate  $\frac{1}{5}$  of the candy right away. How much candy did he eat?

### Question 8.

Obtain an answer to each arithmetic task using an efficient method. Reduce if possible.

- a)  $\frac{1}{2} + \frac{1}{3} + 2 \times \frac{5}{6}$
- b)  $\frac{1}{2} \times \frac{1}{2} + \frac{3}{2} - 2 \times \frac{3}{4}$
- c)  $2\frac{1}{10} + \frac{3}{2} \times \frac{2}{5} - 1\frac{3}{10}$



## MAINTAIN stage

### Mini-task example

Every so often:

- Present two fraction multiplication questions (fraction x fraction and fraction x mixed). Ask students to use the efficient fraction multiplication method to obtain answers. Discuss as needed.

### Rich-task example

Ask students to solve the following problem.

*Three tenths of the planet's surface is land. Half of that land was once forested. What fraction of our planet was once forested? Three quarters of the forested land has now been cleared. What fraction of our planet still is forested?*

### Comments

This is a rich-task because it is a complex problem involving fraction multiplication.